OPTION VALUATION UNDER THE EFFECT OF TIME DILATION

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INTRODUCTION

More than two big theories changed the world in 20th century. But two of them created a completely new framework for science.

ALBERT EINSTEIN
Theory of Relativity

FISCHER BLACK AND MYRON S. SCHOLES
BLACK SCHOLES Option Pricing Model (BLACK SCHOLES Formula)
BASES OF EINSTEINS THEORY

• Theory of Special Relativity 1905
• Theory of General Relativity 1914/1915
• Einstein concluded that in a moving system, time passes more slowly than in a stationary system. ALBERT EINSTEIN called this:

“Moving clocks tick more slowly”
Bases of Einstein's Theory

The Special Theory of Relativity is based on two essential principles:

1. The speed of light is constant in all space-time models.

2. There is no such thing as absolute space or absolute time.

The time dilation can be derived from these two principles. It says that both time and space depend on the observers motion state. If the observer is in the state of a uniform motion, a watch that is moving, is ticking relatively slower.
**BASES OF EINSTEINS THEORY**

The relative time difference between the observer and the object in motion:

\[ \hat{t} = t \sqrt{1 - \left(\frac{v}{c}\right)^2} \]

\( \hat{t} \) = system in motion

\( t \) = stationary system

\( c \) = vacuum speed of light (299,792,458 m/s)

\( v \) = factor speed of light motion relative to stationary system
Bases of Einstein's Theory

• From the perspective of the system in motion, the course of time relative to the stationary system decelerates by:

\[ \sqrt{1 - \left(\frac{v}{c}\right)^2} \]

• The moment \( t \) in the stationary system can be determined like this:

\[ t = \frac{\hat{t}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \]
**Principle of Option Price Calculation**

- In general, the price of an option can be explained as the sum of its *time value* and *intrinsic value*.

- Time value represents the probability that the option will be „*in the money*“ by the end of its term.
PRINCIPLE OF OPTION PRICE CALCULATION

The intrinsic value is the difference between the option strike price $K$ and the underlying price $S$ and can be derived easily.

\[
\text{Intrinsic Value Call} = \max(S - K; 0)
\]

\[
\text{Intrinsic Value Put} = \max(K - S; 0)
\]
PRINCIPLE OF OPTION PRICE CALCULATION

• The financial literature proposes several methods to calculate the price of an option.
• A very common approach to determine the price of an European long call option at time \( t \) was introduced by BLACK AND SCHOLES in 1973.
**BLACK SCHOLES FORMULA**

\[
C(S, t) = S\text{N}(d_1) - Ke^{r(t-t^*)}\text{N}(d_2)
\]

\[
d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2} \sigma^2\right)(t^* - t)}{\sigma \sqrt{t^* - t}}
\]

\[
d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{1}{2} \sigma^2\right)(t^* - t)}{\sigma \sqrt{t^* - t}}
\]

Based on the Black and Scholes formula, the option’s sensitivity to a change in time is

\[
\theta = \frac{\delta C(S, t)}{\delta t}
\]

If all other parameters are constant, the option value changes over time by the value of \(\theta\).
TIME DILATION AND BLACK SCHOLES

• Modulation of two European long call options.
• We assume an underlying price $S$ of 50; strike price $K$ of 50; risk free rate $r$ of 2%; no dividend payments and a volatility $\sigma$ of 30%; the maturity of the earth option to be three years and $\lambda$ (factor of speed of light at which the space ship moves) 0.5.
TIME DILATION AND BLACK SCHOLES

• The figure shows the effect of the speed of the spaceship on the maturity of the space option at earth time.
• The maturity of the space option at earth time increases exponentially up to the speed of light.
• The more the spaceship approaches the speed of light, the more the time in the spaceship decelerates relatively to the time on earth.
TIME DILATION AND BLACK SCHOLES

• For instance if the maturity of the earth option is three years and the spaceship moves with the factor 0.5 of the speed of light, the space option’s maturity at earth time is 3.46 years.

• If the factor is 0.75 it is already 4.54 years.
TIME DILATION AND BLACK SCHOLES

This figure shows the theta value of the earth and the space option (at earth time).
CONCLUSION

• The space option expires significantly later than the earth option.

• The more the speed of the spaceship accelerates, the longer the space options maturity at earth time.

• Hence, the time decay of the space option decelerates with increasing speed of the spaceship.
CONCLUSION

• In the thought experiment mentioned above, the time value elasticity of the space option relative to the earth option is obvious.
• Hence, it can be concluded that the time dilation described by ALBERT EINSTEIN can also be applied to financial options.
• Finally, the time value evolution clearly shows a shift of the time decay in conformity with the patterns described by ALBERT EINSTEIN.
THANK YOU FOR YOUR ATTENTION

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